Axioma In-Practice Series

Risk Resolution: A Framework for Generating Custom Risk Models

Fabien Couderc and Robert Stamicar*

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At Axioma, we often debate what constitutes a standard multi-asset class (MAC) risk model. First, there is the choice of the risk factors. On the one hand, the standard model should consist of a parsimonious number of risk factors, but on the other hand, it should capture all relevant risk factors for a well diversified portfolio. Then there is the estimation of volatility for risk factors. Does a simple EWMA scheme work, or is a more sophisticated GARCH model required? Are the decay factors chosen appropriately? Ultimately, backtests will validate whether risk factor selection and volatility estimates are adequate.

Sometimes, however, a standard risk model is inadequate, especially if the desired granularity from a standard model is not available. For example, a manager trading along a sovereign curve or a manager with significant exposure to covered bonds might require a denser term structure of yields or spreads than a standard model might permit. A framework is needed that allows alignment of risk factors to investment strategies.

Moreover, financial institutions struggle to reconcile gaps between different risk functions across the organization. For instance, under the Fundamental

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Review of the Trading Book (FRTB), banks seeking an internal models-based approach for regulatory capital calculations must perform P&L attribution tests. Designed to show that each desk's actual performance is aligned with the bank's risk models, these tests measure the impact of the differences between risk factors used by front office pricing models and risk factors used by risk management.

In this note, we present a framework called **risk resolution** that bridges different risk functions of an enterprise and allows risk managers to explore transient market behavior. The basic idea is simple: to define relationships between pricing and risk factors. Though a simple concept, risk resolution provides risk managers with finer control of how risk factors translate into pricing factors, reconciles gaps between different risk functions such as the front and middle offices, and allows custom risk models to be tailored to internal processes and investment horizons.

Toward Custom Models

Although risk and portfolio managers agree that measuring and managing risk is essential, they often differ on the best way to achieve this goal. The choice of a risk model is a function of asset coverage and should align with investment strategies. For linear instruments, such as equities, a linear factor model with a parsimonious set of risk factors is appropriate. However, aggregating risk for portfolios that have exposures to fixed income, credit, or derivatives is challenging. When the focus is on asset coverage, a granular approach is commonly employed to capture nonlinear effects. In this case, pricing models that capture asymmetric returns are required, and the number of pricing factors can be large.

Moreover, at the enterprise level, different risk functions analyze risk differently. In fact, among risk functions, investment professionals, and investors, there is not a uniform definition of risk, but rather a matrix of risk definitions. Table 1 lists different risk management requirements across a firm. For example, allocations are performed across different horizons and trading opportunities. At short horizons, tactical and relative value trading requires basis risk modeling, i.e., granular modeling. This contrasts with medium and long horizons, where strategic decision making requires a model with fewer dimensions, i.e., factor-based modeling.

Clearly one view of risk does not satisfy all the risk management requirements (see Table 1). As a result, the multiple views of risk across a firm present the following inherent challenges:

Consistency: A lack of consistency in portfolio modeling can arise in various ways. Modeling the same asset class differently across the firm is problematic. For example, a full repricing (nonlinear) approach can differ significantly from a linear modeling approach, especially for portfolios that include derivatives. Examining tail risk from a value at risk (VaR) approach or stress testing exacerbates these differences.

Communication: The variety of risk analysis impedes communication between risk functions. Whereas sensitivity-based analysis is important for the front office, ex-ante risk measures that capture correlation and concentration are impor-

Table 1: Risk Management Patchwork

Risk Requirements	Comments
Firm management	Firm management's long horizons and board-defined strategies lead to ad-hoc and internal modeling.
Approaches	Different approaches to measuring risk (for instance, factor versus granular modeling) can be a function of asset classes.
Allocations	Allocations are performed across different horizons and trading opportunities.
Investors	Investors need to perform direct and simple comparisons. This requires cross-asset analysis and risk decomposition by asset class.
Alignment of risk and performance	Aligning risk models with investment strategies and performance is challenging.

tant at an enterprise level. To facilitate a dialogue, portfolio and risk managers need to understand and reconcile the differences between sensitivity-based and ex-ante risk, fundamental and quantitative risk drivers, and parametric and simulation-based measures. In addition to examining risk at an aggregate firm level, the risk manager needs to understand how portfolio managers and traders use sensitivity and exposure measures to analyze risk.

Cross-Asset Risk: A limited ability to consider cross-asset class risks is another challenge. Typically different modeling assumptions are used across asset classes, and there is little focus on interactions between assets.

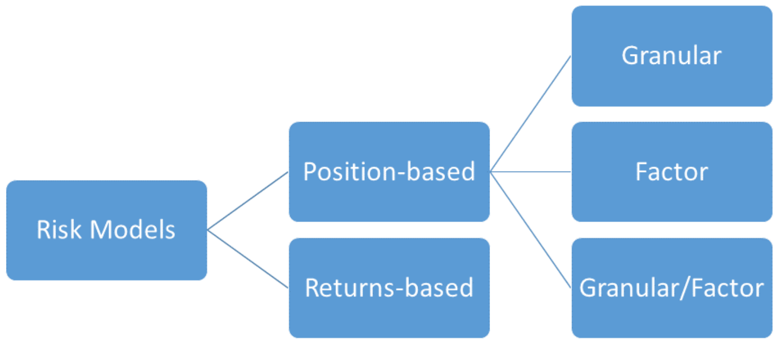
Operational Issues: Operational issues stem from existing rigid approaches. Aggregating risk is difficult when different systems in the firm are used to model different asset classes, each of which comes with its own set of modeling choices.

Growing Complexity in Risk Management

The challenges described above are in part due to the growing complexity in risk. Risk analysis has evolved from notional values weighted by risk weights (as specified in the Basel Accords), to sophisticated frameworks involving full pricing models and portfolio distributional assumptions. We briefly describe the varying levels of complexity below.

Under a *notional/exposures approach*, risk is computed by summing notional or exposure values of securities. For example, for bonds we can compute exposure by present value, but for swaps the underlying notional value is more appropriate since the present value can be close to zero. Under the Basel Accords, risk weights are applied to notionals. This approach is simple and model free. However, it does not handle derivatives, netting or diversification very well.

Figure 1: Risk Model Classification



Another approach to analyzing risk is to compute *sensitivities* to factors such as interest rates or spreads (DV01 and CS01). This is the next level of complexity from notional-based risk measures.[1] Pricing models are required to compute sensitivities, which are in turn used to estimate the change in portfolio value for a small change in risk factor levels. Although useful for analyzing linear hedging, these measures do not capture correlation across risk factors and thus are not useful for capital adequacy calculations.

The most sophisticated frameworks include risk models and stress testing. We can categorize risk models as returns- or positions-based models[2] (see Figure 1). Returns-based modeling is top-down and relies on time series regressions over broad systematic factors to estimate risk. On the other hand, positions-based risk modeling provides risk estimates via a bottom-up approach where each underlying holding is aggregated to the portfolio level. Although returns-based models are fairly straightforward to implement from both a modeling and data standpoint, they suffer from being backward-looking. In contrast, positions-based models incorporate current holdings into their risk estimates and thus provide a forward-looking measure that adjusts as positions are bought and sold. Position-based models also facilitate meaningful aggregations across portfolios and the institution since they incorporate netting and correlations across asset classes.

The level of sophistication of positions-based models can vary significantly. We can broadly categorize these as *factor* or *granular* risk models, which we will describe in more detail. Finally, stress testing analytics usually complement risk-based mod-

[1] The standardized approach under FRTB will be based on sensitivity metrics.

[2] See Jacob and Stamicar [3] for a discussion involving returns- and positions-based modeling.

els, and are invaluable when risk models break down in periods of market turmoil. Stress testing methodologies can include historical, user-defined, and correlated shocks to risk factors. In addition, more advanced techniques include reverse stress testing and utilizing copulas for tail risk.

Factor vs. Granular Risk Models

Table 2 shows how different groups in an organization might estimate market risk. We simplified the risk functions in Table 2 to front and middle offices, but more cross functions can be present in a large organization. Even in this simplified scheme, different asset classes and different investment strategies require different levels of granularity.

Linear factor models measure risk by using a parsimonious set of factors. This has the benefit of greatly reducing the dimensionality of the problem. Instead of explicitly computing asset-by-asset correlations, representing each asset as a linear combination of factors allows us to compute asset correlations by weighting the factor correlations by exposures. More precisely, security returns are decomposed into systematic factors that are common to all assets along with a specific factor. The systematic factors are generally assumed to be normally distributed, allowing for closed-form solutions to risk statistics such as volatility or VaR.

While linear factor models are extremely useful, they do not capture nonlinear payoffs. Institutions such as banks, broker dealers, and hedge funds have exposures not only to equities, but also to fixed income, credit or trading positions employing derivatives. Here the focus is on asset coverage. To aggregate risk across these diverse assets and to capture nonlinear effects, managers typically employ a granular approach. From a risk management perspective, the granular approach consists of two key components:

- **Pricing models**: Pricing models are specified at the asset class level. The payoffs of portfolios comprised of derivatives can differ significantly from the returns of the underlying assets.
- **Pricing factors**: These factors are closely related to the market prices of liquid assets, and they drive the risk-neutral pricing of those assets in pricing models, such as individual equity return time-series, a grid of points from an implied volatility surface, foreign exchange rates, and the term structure of yield curves.

The key goal of a factor model is to reduce the dimension of the covariance matrix of asset returns. On the other hand, the size of the covariance matrix for the granular approach can grow significantly since it is computed from pricing factors (equities, commodities, exchange rates, interest rates, implied volatilities, etc.). In fact the covariance matrix is typically computed on-the-fly since the composition of a multi-asset class portfolio can evolve as positions are bought and sold.

Integration of Factor- and Simulation-Based Approaches

At first, the distinction between factor and granular approaches might seem clear-cut and disjointed. A factor model is typically regarded as equivalent

Table 2: Risk Management across an Enterprise

	Front Office • *Portfolio construction* • *Hedging* • *Performance attribution*	**Middle Office** • *Performance attribution* • *Investor & regulatory reporting* • *Capital allocation*
Equity	Multi-factor models Greeks for derivative hedging	Granular factors Index-based approaches Simulation for derivatives
Fixed Income	Parametric for curve factors Hierarchy of spreads Partial derivatives/linear risk	Full repricing & simulation Many curves Little structure
Credit	Parametric for curve factors Hierarchy of spreads Partial derivatives/linear risk	Many factors Basis risk captured Structural components
Commodities	CMF (const. mat. future) curves	Multi-factor risk models Cross-sectional estimation Simulations for derivative books
Alternatives	Valuation data	Market-related proxies

to a linear model in which risk is computed by applying asset weights and exposures to a covariance matrix of factor returns. In contrast, a granular model is usually applied to a portfolio that contains positions with nonlinear payoffs, and Monte Carlo simulations are deployed for full pricing.

But sometimes the two types of models are best used together. We label this case as "Granular/Factor" in Figure 1. For example, an equity portfolio can often be analyzed with the aid of a factor model. The bets and risks will to some extent be understood in the language of factors. For example, a manager may choose to be overweight *value* or neutralize some particular industry. If the portfolio contains nonlinear securities (such as a put option, perhaps acting as a hedge), then one must deal with a number of additional effects. First, the nonlinear security usually introduces an asymmetry between positive and negative returns. A large positive equity return may have little effect on a put, for example, but a large negative return may significantly increase its value. These nonlinear effects are not captured by a model that assumes all securities are linear. Getting accurate risk numbers usually requires a Monte Carlo simulation with full repricing. Second, nonlinear assets are exposed to risk factors not typically found in equity factor models. For example, an option is exposed to interest rates and implied volatility.

Thus, to combine a linear factor model with Monte Carlo simulations, we need to address both consistency and pricing factors:

- **Consistency:** There is no tension between using a factor model and simulation. We directly simulate the factor model, and thus enable risk reporting that decomposes risk along familiar factor lines. We also incorporate various nuances of a well-estimated factor model, such as different decay factors for volatility, correlation, and specific risks, as well as Newey-West estimators to dampen autocorrelation effects. In fact, *what* one simulates should be chosen by the user. For example, in effect, we have just described mapping equities to a factor model, but other choices are possible. For instance, a user could map equities to a dense model (where each asset is a separate risk factor), albeit at the cost of losing the factor decomposition of risk.
- **Pricing factors:** Any additional factors, such as pricing factors, are simulated (with correct correlations) along with the parsimonious factors from the risk model. In the case of an option, several nodes from an interest-rate curve, as well an implied volatility factor, would thus be simulated.

Risk Resolution—Prelude

We just described how factor models can be combined with a Monte Carlo simulation framework that utilizes nonlinear pricing models. Moreover, simulation with full repricing can be accompanied by a factor model risk decomposition.

Although combining factor models with Monte Carlo simulations might now seem straightforward, we still must consider key modeling choices in this integrated framework. First we chose risk factors (fundamental factors, pricing factors such as interest rates and implied volatility), pricing models (for example, Black-Scholes for options, linear factor model for equities), and linear or nonlinear repricing. In addition, there is the specification of risk settings around volatility and correlation estimations, and distribution assumptions. These settings are important to align risk estimations with investment strategy horizons.

Thus, even when we know how to combine factor model and Monte Carlo simulations, we still need the ability to decompose modeling choices along dimensions of risk factor selection and pricing model decisions. The risk resolution framework gives us that ability. Risk resolution also provides a framework for reconciling and analyzing multiple views of risk within an organization. In the next section, we describe this framework.

Risk Resolution Framework

To make sense of the complexity and different risk views a firm needs to analyze and reconcile, we introduce the concept of *risk resolution*:

> *Risk resolution defines the way changes in risk factors are transmitted to pricing factors.*

Recall that pricing factors are fine-grained factors, such as volatility surfaces, individual equity time series, and term structures of interest rates. They are closely aligned with pricing models. On the other hand, risk factors from factor models are more parsimonious and explanatory of the underlying sources of risk. They drive the movement of the pricing factors in risk decomposition, simulation, or stress testing. The distinction between risk factors and pricing factors can be blurry, but we will provide examples in the next section to clarify these differences.

Risk resolution provides a formal separation between risk factors and pricing factors. The concept of risk resolution is depicted in Figure 2. Risk resolution is simply a mapping between risk factors and pricing factors; it specifies how risk factors are related to pricing factors and *vice versa*. Although risk resolution is a mapping, the flexibility of *explicitly* defining the relationship between factors is the essence of the framework. In summary, risk resolution:

- Provides finer control of how risk factors translate into pricing factors
- Allows custom risk models to be tailored to internal processes and investment horizons
- Reconciles gaps between different risk functions within an organization, such as front and middle office

Risk Models and Risk Resolution

We can succinctly represent Figure 2 as

$$\boxed{F \overset{\phi}{\mapsto} P \overset{\theta}{\mapsto} V} \tag{1}$$

where F represents risk factors, P represents pricing factors, V represents asset prices, ϕ represents a given risk resolution mapping, and θ represents pricing models. Together, we refer to the risk resolution mapping and the choice of pricing models as the **risk resolution framework**.

Note that (1) incorporates the choice of risk factors (and asset pricing models). This may seem trivial, but is worth highlighting. For example, selecting fundamental equity factors is usually synonymous with selecting a particular fundamental equity model. For fixed income assets, we might

Figure 2: Risk Resolution Framework

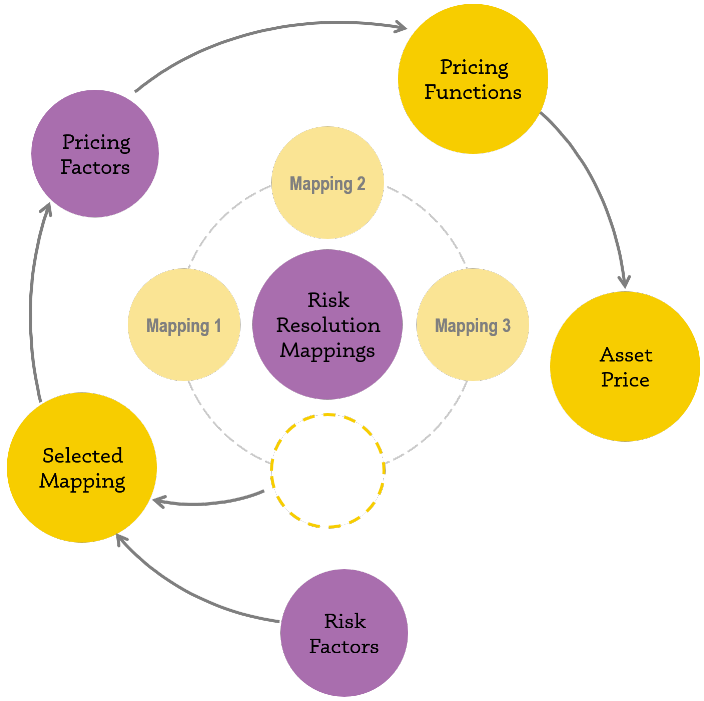
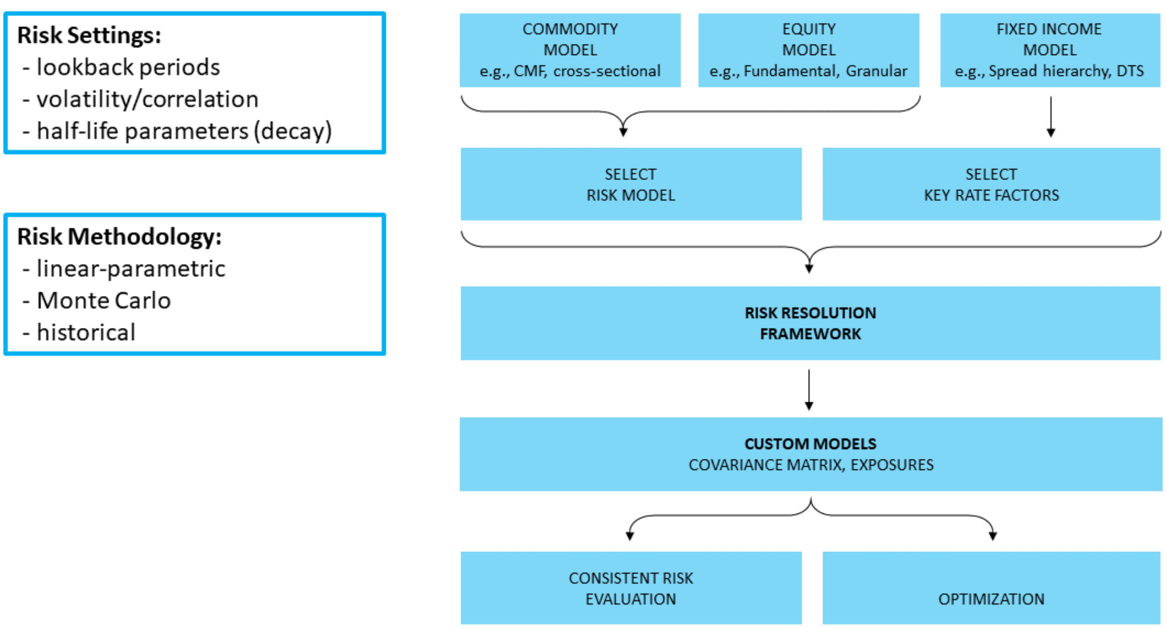


Figure 3: Aggregation of Risk Models



limit the number of key rates to control the number of risk factors. Risk resolution allows aggregation of different asset class risk models. This is useful in the context of custom model generation. Figure 3 is another depiction of risk resolution in the context of custom model generation. Later we will discuss waterfall structures under risk resolution.

For a risk model, the final piece required is a specification of the risk dynamics, which include the distributional assumptions, forecast horizon, aging assumptions, and covariance estimation of risk factor returns. Thus the specification of risk resolution mapping, pricing models, and risk dynamics defines a risk model.

Casting Factor and Granular Models

We can cast both the traditional factor and granular models in terms of risk resolution and pricing models. First, consider a linear factor model where the future prices are modeled as

$$\begin{aligned} P_1(t+\Delta t) &= P_1(t)(1+r_1(\Delta t)) \\ P_2(t+\Delta t) &= P_2(t)(1+r_2(\Delta t)) \\ &\vdots \\ P_n(t+\Delta t) &= P_n(t)(1+r_n(\Delta t)) \end{aligned}$$

where P_i is the value of asset i and r_i is the return of asset i over the time horizon Δt. We can compactly write the equations above in matrix notation as

$$P_{t+\Delta t} = D_t(1+r) \tag{2}$$

where D_t is a diagonal matrix with entries $P_i(t)$. Under a linear factor model, the returns are given by

$$r = Xf + \varepsilon \tag{3}$$

where f is the factor returns, X is the exposure, and ε is the specific returns. Under our risk resolution framework, the pricing factors are the same as the asset values. In terms of mappings, we have the following:

$$\phi(f) = D_t(1 + Xf + \varepsilon) \tag{4}$$

$$\theta = I \tag{5}$$

where I is the identity mapping.

Under a granular approach, the risk factors and pricing factors are identical ($\phi = I$). In addition, the pricing function is the identity mapping ($\phi = \theta$). The emphasis on asset coverage translates into asset-specific pricing functions.

Sample Risk Resolution Mappings

In this section, we list some examples involving risk resolution mappings.

- **Equity:** Under a factor model, the risk resolution mapping is a linear combination of fundamental factors such as momentum, value, and growth, and the pricing function is the identity (see (4)–(5)). Under a granular model, the risk factor is the equity itself. Both the risk resolution and pricing mappings are the identity ($\phi = \theta = I$); see Table 4.

- **Equity Options:** Consider a portfolio of equity options of varying strikes and tenors. The pricing function θ is the Black-Scholes formula for call options:

$$C = S\Phi(d_1) - Xe^{-rT}\Phi(d_2)$$

$$d_{1,2} = \frac{\log(S/X) + (r \pm \sigma^2/2)T}{\sigma\sqrt{T}}$$

How complex are Risk Resolution Mappings?

The risk resolution mapping $F \overset{\phi}{\mapsto} P$ can vary in complexity:

Linear mapping:

- Granular models approach
- Linear factor models
- Beta-projection, e.g., interest rate term structures, volatility surfaces

Nonlinear mappings:

- Credit spread translations, e.g., CDS spreads to bond issuer spreads
- Structural models

where S is the equity price, X is the strike, r is the spot interest rate, σ is the equity volatility, and T is the time to maturity.

Consider the following parsimonious risk resolution with the following pricing and risk factors:

Pricing Factors:

– Equity price itself
– All nodes (strike, expiry) on an implied volatility surface
– All nodes on the risk-free curve

Risk Factors:

– Equity factors (fundamental model)
– Implied Volatility: 6m expiry, 50 delta
– IR curve: 1m, 6m, 1y, 2y, 5y, 10y, 30y

Note that this risk resolution will significantly reduce the total number of risk factors for the same underlying issuer or index since we do not use all the nodes from the implied volatility surface.

Alternatively, under granular modeling, the risk factors would instead represent all the pricing factors listed above. i.e., the risk resolution mapping ϕ is the identity mapping. See Table 4.

- **Merton Model for AT1 bonds:** In contrast to the examples above, the risk resolution mapping can be nonlinear. Consider the spread risk of an AT1 (Additional Tier 1) bond. Since AT1 bonds are hybrid securities that sit lower in the capital structure than other debt, it is reasonable to model them via equity factors.

  Under a Merton or structural model, we can translate equity returns into hazard rates or spreads, which in turn can reprice the bond. We can represent the mapping as follows:

$$(S, \sigma_s, r) \overset{\phi}{\mapsto} h$$

  where the stock price S, stock volatility σ_s, spot rate r are risk factors, and the spread h is the pricing factor.

  As a concrete example, consider the following

Merton-like model implementation:

$$h = -\frac{1}{t}\log Q$$

where the survival probability Q is given by:

$$Q(S, \sigma_s, r; S^\star, t) = \Phi\left(\frac{\mu t - \log(\frac{S^\star}{S})}{\sigma_s\sqrt{t}}\right) - \left(\frac{S^\star}{S}\right)^{\frac{2\mu}{\sigma_s^2}} \Phi\left(\frac{\mu t + \log(\frac{S^\star}{S})}{\sigma_s\sqrt{t}}\right)$$

where $\mu = r - q - \sigma_s^2/2$, q is the continuous dividend, $S^\star$ is the equity default barrier, and Φ is the cumulative normal distribution.[3] Clearly, this is a nonlinear mapping of risk factors to a pricing factor.

Moreover, instead of using equity prices as risk factors, we can use risk factors from a fundamental equity model. Table 3 provides an example of such a risk resolution for a sample portfolio comprised of AT1 bonds.

Aggregating Risk Models and Risk Resolution Waterfalls

In a multi-asset class setting, a global model can be constructed by aggregating different asset class risk models. Figure 3 depicts the steps required to define a global model: (i) risk setting configuration, (ii) risk methodology, and (iii) risk model aggregation. In this last step, risk resolution comes into play under the specification of a mapping between risk and pricing factors within each asset risk model. For example, Axioma employs a fixed-income model based on a hierarchy of spread curves. In this case, risk resolution can be used to restrict the number of key rate factors or collapse parts of the spread hierarchy to produce a more parsimonious model. Clearly, the precise aggregation of models is important, and backtesting should be performed on portfolios to verify that the aggregated model is providing accurate ex-ante risk statistics.

As an example, Table 5 displays Axioma's Equity Worldwide Model and an aggregated equity risk model side by side. The aggregated model, which is based on country models, is defined via a waterfall process. First, each equity is mapped to its corresponding country model. If a country model is not available, the equity is mapped to a regional model, else to the Worldwide Model. This model might be more appropriate for portfolios that have a high exposure to a handful of countries, rather than a globally diversified portfolio. The overall risk is different between the two risk models in Table 5. As expected, the risk factors also differ. For example, consider Japanese stocks in Table 5. Under the Equity Worldwide Model, we have a market factor and a country factor, which by design are not present under the waterfall risk resolution. In a similar fashion, we can define other risk resolutions based on different waterfalls.

[3] In fact, this is an equity barrier model that is similar to Merton models based on knock-out barriers. See Stamicar [5] for more details.

Table 3: Risk Resolution via Nonlinear Mapping

Reporting Levels	Present Value USD	Standalone Vol USD	Marginal Vol USD
CoCo (AT1) Portfolio Apr 02 2015	98 615.90	367.37	367.37
▸ ALDERMORE GROUP PLC - REG S Perpetual (1)	1668.26	6.67	4.34
▾ ALLIED IRISH BANKS PLC - 10% COCO 2016 (1)	1206.46	8.02	4.13
▾ Risk Type : Issuer Credit (5)		6.33	0.66
▸ Country (1)		1.61	−0.61
▸ Industry (1)		0.35	0.04
▸ Market (1)		1.09	0.49
▸ Specific Risk (1)		5.85	0.09
▸ Style (9)		0.89	0.34
▸ Risk Type : FX (1)		5.50	3.45
▸ Risk Type : Interest Rate (3)		0.19	0.02
▸ Risk Type : Vega (1)		0.01	0.01
⋮	⋮	⋮	⋮

Spread modeled via equity factors

Table 4: Sample Risk Resolution and Pricing Mappings

Risk **Resolution**	**Risk Factors** F	**Factor Mapping** ϕ	**Pricing Factors** P	**Pricing** θ
Equity *Granular Model*	Equity (same as P)	Identity	Equity	Identity
Equity *Factor Model*	Fund. factors F_E	Lin comb of F_E	Equity	Identity
Equity Option *Granular Model*	Same as P	Identity	Equity All (strike, tenor) nodes All discount factors	Black-Scholes
Equity Option *Factor Model*	Fund. factors F_E (6m, 50 delta) (6m,1y,5y,10y) IR nodes	Lin comb of F_E Interpolation of vol surface Interpolation of IRs	Equity All (strike, tenor) nodes All discount factors	Black-Scholes
HY Corporate Bond *Factor Model*	Fund. factors F_E (6m, 50 delta) (6m,1y,5y,10y) IR nodes	Nonlinear comb of F_E Interpolation of vol surface Interpolation of IRs	Hazard rates All (strike, tenor) nodes All discount factors	Hazard rate pricing

Reconciling Differences via Factor Selection and Projections

One key theme in this note has been that risk resolution can reconcile multiple risk views for the same portfolio. In the previous section, we provided an example where two different risk models could be viewed side by side, and we used it to identify differences in risk factor decompositions. In this section, we will provide examples involving multiple risk model views, varying factor selection under the same pricing models, and how a risk resolution based on beta-projections can reconcile differences from a parsimonious and granular model.

For an example of the way risk resolution can help to reconcile risk views, consider an equity portfolio comprised of US and Canadian stocks with single-name put options. In Table 6, we present a risk decomposition report with three different risk resolutions (and models). For instance, the Global Model (last column) might represent the model that is used to aggregate portfolios at the enterprise level, while the remaining columns represent different risk views within different groups of the organization. For example, since this is a USD/CAD based portfolio, the portfolio manager might prefer to use a fundamental model that is based on North American factors instead of global factors (column labeled "Regional"). In the second column, we use granular factors with equities, perhaps from another group in the firm that does not have access to equity fundamental models.

Note that the risk decompositions differ. Under "Risk Type: Equity," we see the typical fundamental factor breakdown, such as country, style, and specific risk. Under the granular approach, each equity is its own risk factor; the cell labeled "Stock" represents the granular risk.

Our next example involves risk factor selection under the same pricing model. Here we will focus on vega risk for equity options. Consider Table 7, where the risk decomposition of a call and put equity for Tesla stock is displayed. One key risk factor is vega risk (the vol of vol). In this particular risk resolution we consider only one node on the volatility surface, the 6-month 50-delta node. Effectively, we are capturing only parallel shifts of the volatility surface.

The beginning of 2017 was an interesting period where Tesla calls have priced at a higher volatility premium relative to puts, driven by the high demand for Tesla call options. In addition, the skew was significant over this period. See Figure 4, where we observe a volatility skew of 3%. Thus a different risk resolution incorporating the skew would give a greater ex-ante risk prediction. Comparing risk views is useful in accessing how different risk factors can impact risk estimates, and regulatory bodies are beginning to require that mismatches between the front and middle offices are minimal. Notably, the *weak* version of the P&L attribution test under FRTB attempts to quantify these differences in risk factor selection. Front office pricing analytics incorporate more risk factors and are generally more accurate than enterprise risk systems. The weak version of the P&L attribution examines the P&L that would be produced by the bank's pricing models if they included only risk factors used in risk management models. (See Wood [6].)

Our last example involves a portfolio comprised of

Table 5: Risk Resolution Waterfall

Reporting Levels	Present Value USD	Worldwide Model USD	Country Models via Waterfall USD
Equity - World	336 582 035	3 687 965	3 465 363
⋮	⋮	⋮	⋮
▸ HK (2)	4 433 018	28 536	27 397
▸ IE (2)	471 133	5064	5461
▸ IT (2)	1 921 599	27 442	31 565
▾ JP (2)	30 873 080	147 831	164 115
▾ Risk Type : Equity (5)		237 511	254 177
▾ Country (1)		−89 632	
Japan		−89 632	
▸ Industry (46)		6240	221 253
▾ Market (1)		287 895	
Global Market		287 895	
▸ Specific Risk (103)		1215	826
▸ Style (11)		31 794	32 098
▸ Risk Type : FX (1)		−89 680	−90 062
▸ LU (2)	9 247 216	74 820	75 664
▸ NL (2)	3 776 702	39 012	43 271
▸ NO (2)	567 945	7096	7793
▸ SE (2)	3 155 200	39 460	45 347
⋮	⋮	⋮	⋮

Table 6: Multiple Risk Views Of Standalone VaR With Different Risk Resolutions

Reporting Levels	**Granular VaR95 USD**	**Regional VaR95 USD**	**Global VaR95 USD**
US/CA Equity & Options Feb 17 2016	161 227	144 498	150 868
▾ American Equity Option (3)	10 220	11 375	11 004
▸ Risk Type : Equity (6)	8307	9015	8687
▸ Risk Type : Interest Rate (3)	26	52	52
▸ Risk Type : Vega (1)	2982	3558	3558
▾ Stock (1)	170 410	155 288	160 148
▾ Risk Type : Equity (6)	170 410	155 288	160 148
▸ Country (2)		6975	54 407
▸ Industry (79)		7555	9595
▸ Market (2)		149 132	148 928
▸ Specific Risk (504)		13 716	13 926
▸ Stock (504)	170 410		
▸ Style (9)		10 015	17 434

US government bonds and US agency bonds. Table 8 displays different one-day VaR measures at the 95% confidence level for a portfolio of US government bonds and US agency bonds. All VaR statistics are computed as risk contributions in this report. The column labeled "VaR95" represents our standard setting where seven key rate factors are employed. This portfolio has a duration of 3.5 years, and as expected the bulk of the interest rate contribution is centered around the portfolio's duration. The column "VaR95 (Projection)" is derived from a risk resolution where only one key rate is selected, the 5y node. This risk resolution might be suitable for an asset allocation model, in which a parsimonious number of risk factors are required, and in which parallel shifts are sufficient. Perhaps a risk function utilizes an optimizer for asset allocation and requires a parsimonious number of risk factors.

It is no surprise that the risk estimates in Table 8 vary, and that the risk arising from the 5y node is overestimated. We can explain this observation as follows. The volatility of key rates with a tenor greater than one are similar, while the volatility of key rates with a tenor less than one are significantly lower. And given that the portfolio duration is less than 5 years, risk will be overestimated using the 5y node.

We can reconcile the difference in risk estimates by utilizing the full-term structure via betas. Column "VaR95 (Beta-Projection)" displays risk contributions across each node, but all the risk is still driven off the 5y node. Here we observe that the risk estimates are much more in line with the granular version that utilizes the term structure.

Table 7: Risk Report for Tesla Options

Reporting Levels	Present Value USD	Standalone Vol USD	Marginal Vol USD
Tesla Options Jun 30 2017	13 792.50	440.89	440.89
▾ TSLA 190118C300000 (1)	10 225.00	532.98	522.26
▾ American Equity Option (3)	10 225.00	532.98	522.26
▸ Risk Type : Equity (5)		534.80	513.10
▸ Risk Type : Interest Rate (2)		5.58	0.37
▾ Risk Type : Vega (1)		89.27	8.79
▸ Vol Surface: TESLA INC (1)		89.27	8.79
▸ TSLA 190118P280000 (1)	3567.50	133.94	−18.37

Figure 4: Volatility Skew for Tesla

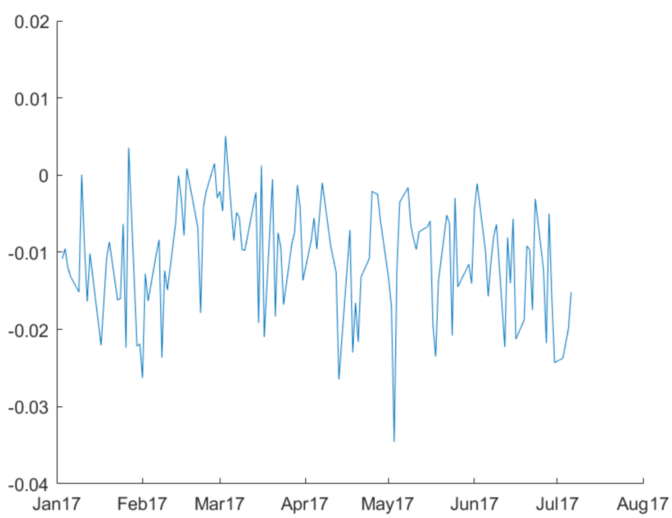


Table 8: Projections Using Risk Resolution for a Fixed-Income Portfolio

Reporting Levels	Present Value USD	VaR95 USD	VaR95 Projection USD	VaR95 Beta-Projection USD
US Government Bonds	186 407 005.80	291 925.61	383 140.68	291 350.01
▾ Risk Type: Interest Rate		292 372.42	378 346.02	282 826.89
US.USD.GVT.ZC : 1M		−1.51		−6.84
US.USD.GVT.ZC : 6M		77.51		48.03
US.USD.GVT.ZC : 1Y		2286.67		2273.12
US.USD.GVT.ZC : 2Y		26 560.42		45 179.83
US.USD.GVT.ZC : 5Y		164 072.53	378 346.02	145 729.41
US.USD.GVT.ZC : 10Y		46 727.11		35 138.63
US.USD.GVT.ZC : 30Y		52 780.46		54 536.71
▾ Risk Type: Credit Spreads		−447.96	4804.13	8536.82
USD.AGNCY.FHLB		−1205.25	2298.45	1936.87
USD.AGNCY.FHLMC		757.29	2505.68	6599.95

Conclusion

In this note we presented the risk resolution framework, which bridges different risk functions of an enterprise and allows risk managers to explore transient market behavior. The risk resolution framework consists of the mapping between risk and pricing factors, along with the asset pricing functions. Together with risk dynamic settings, such as look-back periods and marginal distribution assumptions, custom models can be specified.

Ultimately, risk resolution does not hard code modeling choices for risk professionals. Instead, it allows one to select appropriate risk factors and pricing functions upfront, bringing flexibility in how custom multi-asset class models are generated.

References

[1] Axioma (2013). Risk model machine: customizable risk models tailored to your investment process, Axioma Product Note.

[2] Couderc, F. (2012). Risk resolution: the missing element, Axioma presentation.

[3] Jacob, P. and Stamicar, R. (2014). Tailoring risk modeling for hedge fund investors, Risk.net.

[4] McMurray, J. and Melnikov, S. (2013). Measuring market risk: approaches and inherent assumptions, *The Journal of Investing*, 22(1), 49–56.

[5] Stamicar, R. (2016). CoCo risk: practical approaches to measuring risk, Axioma Research Paper No. 72.

[6] Wood, D. (2016). The P&L attribution mess, Risk.net.

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United States and Canada: +1-212-991-4500

Europe: +44-207-856-2424

Asia: +852-8203-2770

Sales: sales@axioma.com

Customer Support: support@axioma.com